An example of when you might want polynomial long division

$$\lim_{x\to 6} \frac{(x-6)(x+7)}{4x^5-22x^4-15x^3+18x^2+x-6}$$ → this is $=0$ at $x=6$ (Check this)

- we have a $\frac{0}{0}$ situation, so we need to do more work: we probably want to factor an $(x-6)$ out of the denominator

$$\begin{array}{r|l}
 & 4x^4+2x^3-3x^2\quad +1 \\
x-6 & 4x^5-22x^4-15x^3+18x^2+x-6 \\
 & \underline{4x^5-24x^4} \\
 & +2x^4-15x^3+18x^2+x-6 \\
 & \underline{2x^4-12x^3} \\
 & -3x^3+18x^2+x-6 \\
 & \underline{-3x^3+18x^2} \\
 & x-6 \\
 & \underline{x-6} \\
 & 0
\end{array}$$

Rewrite our limit

$$\lim_{x\to 6} \frac{(x-6)(x+7)}{(x-6)(4x^4+2x^3-3x^2+1)} = \lim_{x\to 6} \frac{x+7}{4x^4+2x^3-3x^2+1} = \frac{13}{5509}$$

you've seen that $\lim_{u \to 0} \frac{\sin u}{u} = 1$

A simple example:

$$\lim_{x \to 0} \frac{\sin 3x}{x}$$

This is close to what we know, but not quite (we have 3x inside the sine but only an x in the denominator)

multiply and divide by 3 ($\frac{3}{3} = 1$)

$$= \lim_{x \to 0} \frac{3}{3} \frac{\sin 3x}{x}$$

$$= \lim_{x \to 0} 3 \cdot \frac{\sin 3x}{3x}$$

$$= 3 \lim_{x \to 0} \frac{\sin 3x}{3x} \quad \text{(Limit Theorem)}$$

$$= 3 \cdot 1 = 3 \quad \lim_{u \to 0} \frac{\sin u}{u} = 1$$

Example 2:

$$\lim_{x\to 0} \frac{\tan 3x}{\sin 7x}$$

$\tan u = \frac{\sin u}{\cos u}$

$$= \lim_{x\to 0} \frac{\sin 3x}{\cos 3x} \frac{1}{\sin 7x}$$

- rewrite

$$= \lim_{x\to 0} \frac{1}{\cos 3x} \frac{\sin 3x}{1} \frac{1}{\sin 7x}$$

- $\frac{\sin 3x}{1}$ and $\frac{1}{\sin 7x}$ almost look like what we want. we'll use a similar trick: multiply the whole thing by $\frac{x}{x} = 1$ and rearrange

$$= \lim_{x\to 0} \frac{1}{\cos 3x} \frac{\sin 3x}{1} \frac{1}{\sin 7x} \left(\frac{x}{x}\right)$$

$$= \lim_{x\to 0} \frac{1}{\cos 3x} \frac{\sin 3x}{x} \frac{x}{\sin 7x}$$

$$= \lim_{x\to 0} \frac{1}{\cos 3x} \frac{\sin 3x}{x} \frac{1}{\frac{\sin 7x}{x}}$$

we can flip a fraction this way

- Now we have $\frac{\sin 3x}{x}$ and $\frac{\sin 7x}{x}$ which look like our first example

$$= \lim_{x\to 0} \frac{1}{\cos 3x} \frac{3}{3} \frac{\sin 3x}{x} \frac{1}{\frac{7}{7} \frac{\sin 7x}{x}}$$

$$= \lim_{x\to 0} \frac{1}{\cos 3x} \cdot 3 \cdot \frac{\sin 3x}{3x} \cdot \frac{1}{7 \cdot \frac{\sin 7x}{7x}}$$

$\frac{\sin 7x}{7x} \to 1$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$1 \cdot 3 \cdot 1 \quad \frac{1}{7}$$

- Limit theorem:
  $\lim f \cdot g = (\lim f)(\lim g)$
  ie. look at each factor

$\cos(3x) \to 1$
as $x \to 0$
(because $\cos(0) = 1$)

$$= \frac{3}{7}$$